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On the performance of cooperative systems with distributed linear block coding

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ABSTRACT

Small-scale fading is one of the main problems in wireless communication systems. Multiple transmit/receive antennas, providing spatial diversity, are a common solution to combat fading, but practical constraints at the user location may limit their use. User cooperation is an efficient technique to introduce spatial diversity when multiple antennas are not suitable. In this paper we study the physical-layer performance of a cooperative system based on distributed linear block coding. Analytical results in terms of bit error rate and outage probability are presented when perfect decoding at the user location is assumed. Simulation results in terms of bit error rate are shown, taking into account the impact of errors on decoding and channel estimation at both the user location and the receiver location. Two scenarios are considered, representing uplink communications from static users to a static or mobile base station.

1. Introduction

Wireless communication systems experience large variations in the received signal strength as a function of the relative positions among the transmitter, the receiver, and the scatterers contributing to the multi-path propagation of radio waves. This phenomenon, referred to as small-scale or multi-path fading, severely degrades the system performance; thus numerous techniques have been developed to attack it. Diversity is one the most powerful techniques against fading [1,2], and is often deployed into the space, time and frequency domains.

Spatial diversity is achieved by means of multiple antennas at the transmit/receive side, and it is commonly used in conjunction with space–time codes [3,4]. However, user constraints may not allow the use of multiple antennas; thus cooperation has recently emerged in order to provide spatial diversity while using single antennas at the user location [5,6]. The key idea is that users share their single antennas, transmitting information on behalf of other users as well as their own information. Antenna sharing comes almost for free, due to the intrinsic nature of the wireless channel. It is worth noticing that this differs from the classical relay scenario [7], since users behave both as sources and relays.

Various cooperative schemes have been proposed recently, for example, *amplify-and-forward* (resp. *decode-and-forward*), in which users listen, amplify (resp. decode), and transmit signals from their partners [8,9]. *Coded cooperation* [10–12] is more efficient as channel coding replaces repetition: codewords of each user propagate through different channels. More advanced schemes are based on signal superposition [13] and on code superposition [14,15]. A cooperative algorithm deploying distributed channel coding with linear block codes was proposed in [16]: the symbols transmitted over the channel are a combination of the source symbols (coming from the user) and the side symbols (coming from the cooperative partners).

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This paper builds upon the technique in [16] in order to provide a practical scheme for coded cooperative communications by means of Distributed Linear Block Coding (DLBC). Users are required to transmit each on a different orthogonal channel and listen to the information sent on the remaining ones. The scheme is meant to operate in quasi-static scenarios, in which the block-fading assumption represents the worst case in terms of diversity provided by the channel. Communications happen through transmission of packets of symbols, and Channel State Information (CSI) at the receiver side is obtained via linear Minimum Mean Square Error (MMSE) channel estimation [17]. The transmission scheme is composed of two steps: (i) non-cooperative transmission of pilot symbols in order to perform channel estimation; (ii) cooperative transmission of data symbols iteratively applying the DLBC algorithm.

The contributions of this paper are the following:

(i) after recalling from [16] the Bit Error Rate (BER) performance, diversity results are confirmed in terms of Outage Probability (OP) to strengthen the fact that the method which the proposed scheme relies on provides diversity;
(ii) the BER performance of a practical scheme using DLBC is analyzed via numerical simulations.

By a practical scheme, we mean that we consider here (differently from [16]) channel estimation at both the user location and the Base Station (BS). Also, we here assume packet-based transmission (as usually assumed in modern systems), and in order to implement channel estimation, packets include both data symbols and pilot symbols. The effect of the Pilot Percentage (PP) in the packet is studied via simulations. More specifically, two scenarios have been considered with different assumptions on the Signal-to-Noise Ratio (SNR), representing a Cooperative Group (CG) with static users and moving BS, and a CG with static users and static BS.

Notation- Bold upper-case (resp. lower-case) letters denote matrices (resp. column vectors); $\hat{a}$ denotes an estimate of a; $(\cdot)^{\mathrm{T}}$ denotes transpose.

2. Summary of distributed linear block coding

DLBC [16] refers to a CG with J users assumed synchronous on symbol time (ST). The effects of synchronization errors are beyond the scope of this paper, while ST synchronization techniques my be found in [18]. Each user transmits K symbols over N STs at rate $R = K/N$ by means of a linear block code with minimum distance $d_{\min}$. The simple case with $J = 3$ users cooperating at rate $R = 2/3$, already described in [16] and referred to as *the basic example*, is sufficient to understand the mechanism of DLBC and its performance in terms of BER and OP.

2.1. Transmission and reception scheme.

Assume each user has two source bits to send to the BS by use of three code bits, namely $\{b^{(j)}(1), b^{(j)}(2)\}_{j=1,2,3}$, and $\{c^{(j)}(1), c^{(j)}(2), c^{(j)}(3)\}_{j=1,2,3}$. Each user transmits on a different orthogonal channel and adopts BPSK modulation, i.e. the following one-to-one mapping from bits to symbols:

$$x^{(j)}(n) = \mathrm{BPSK}(c^{(j)}(n)) = 2c^{(j)}(n) - 1.$$

Assuming a circular ordering among users, each user acts as follows: (i) the symbol to be sent in the first ST corresponds to its own first source bit; (ii) the symbol to be sent in the second ST corresponds to the sum between its own second source bit and the estimated bit from the first ST transmission of the previous user; (iii) the symbol to be sent in the third ST corresponds to the sum between the estimated bit from the second ST transmission of the previous user and the estimated bit from the first ST transmission of the twice-previous user. The sum between bits is meant to be an XOR-sum. The transmitted bits are

$$\begin{aligned} c^{(j)}(1) &= b^{(j)}(1) && \text{in the 1 st ST} \\ c^{(j)}(2) &= b^{(j)}(2) + \hat{c}^{(j-1)}(1) && \text{in the 2 nd ST} \\ c^{(j)}(3) &= \hat{c}^{(j-1)}(2) + \hat{c}^{(j-2)}(1) && \text{in the 3 rd ST,} \end{aligned}$$

where $j-1$ and $j-2$ refer to the circular ordering of the users. Diversity is obtained because the symbols travel through different orthogonal channels.

Assuming perfect decoding at user location, the DLBC at the BS is described in terms of bits as $\boldsymbol{c} = \boldsymbol{B}\boldsymbol{b}$, where $\boldsymbol{b}^{\mathrm{T}} = (b^{(1)}(1), b^{(2)}(1), b^{(3)}(1), b^{(1)}(2), b^{(2)}(2), b^{(3)}(2))$, $\boldsymbol{c}^{\mathrm{T}} = (c^{(1)}(1), c^{(2)}(1), c^{(3)}(1), c^{(1)}(2), c^{(2)}(2), c^{(3)}(2), c^{(1)}(3), c^{(2)}(3), c^{(3)}(3))$, and

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix};$$

and may be decomposed into three subcodes $\boldsymbol{c}_{(i)} = \boldsymbol{A}\boldsymbol{b}_{(i)}$, $\boldsymbol{c}_{(ii)} = \boldsymbol{A}\boldsymbol{b}_{(ii)}$, $\boldsymbol{c}_{(iii)} = \boldsymbol{A}\boldsymbol{b}_{(iii)}$, where $\boldsymbol{b}_{(i)}^{\mathrm{T}} = \left(b^{(1)}(1), b^{(2)}(2)\right)$, $\boldsymbol{c}_{(i)}^{\mathrm{T}} = \left(c^{(1)}(1), c^{(2)}(2), c^{(3)}(3)\right)$, $\boldsymbol{b}_{(ii)}^{\mathrm{T}} = \left(b^{(2)}(1), b^{(3)}(2)\right)$, $\boldsymbol{c}_{(ii)}^{\mathrm{T}} = \left(c^{(2)}(1), c^{(3)}(2), c^{(1)}(3)\right)$, $\boldsymbol{c}_{(iii)}^{\mathrm{T}} = \left(c^{(3)}(1), c^{(1)}(2), c^{(2)}(3)\right)$, $\boldsymbol{b}_{(iii)}^{\mathrm{T}} = \left(b^{(3)}(1), b^{(1)}(2)\right)$, and $\boldsymbol{A}^{\mathrm{T}} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. The minimum distance of the code is $d_{\min} = 2$.

Such a decomposition is exploited at the receiver in order to save computational complexity. The signal received at the BS from the jth user on the nth ST is modeled as

$$r^{(j)}(n) = \sqrt{R\mathcal{E}_b}\, h^{(j)} x^{(j)}(n) + w^{(j)}(n), \tag{1}$$

where $\mathcal{E}_b$ is the energy per source bit, $h^{(j)}$ is the channel gain, and $w^{(j)}(n)$ is the additive white Gaussian noise. Collecting $\left\{r^{(j)}(1), r^{(j)}(2), r^{(j)}(3)\right\}_{j=1,2,3}$, the BS recovers the source information referring to the previous subcodes, with ML decoding performed via computation of 12 distances in a 3-dimensional space instead of 64 distances in a 9-dimensional space.

2.2. Bit error rate and outage probability

Cooperation introduces spatial diversity into the system, thus making communications more reliable. The beneficial effect of cooperation may be viewed both in terms of BER and OP. Both curves (plotted against the SNR in the log–log domain) present an increase of the negative slope w.r.t. the case without cooperation.

Assuming Rayleigh fading channels, in the absence of cooperation, the BER and the OP are

$$P_e(\Gamma) \approx \left(\frac{2^K-1}{4}\right)\left(\frac{1}{1+\Gamma}\right), \tag{2}$$

$$P_o(\Gamma) \approx \frac{2^R-1}{\Gamma}, \tag{3}$$

respectively, while cooperation with DLBC replaces Eqs. (2) and (3) with

$$P_e(\Gamma) \approx \left(\frac{2^K-1}{4}\right)\left(\frac{1}{1+\Gamma}\right)^{d_{\min}}, \tag{4}$$

$$P_o(\Gamma) \approx \left(\frac{2^R-1}{\Gamma}\right)^{d_{\min}}, \tag{5}$$

respectively, where $\Gamma = 2R\mathcal{E}_b/\eta_o$ is the average SNR at the BS, defined as the ratio between the average received energy per source symbol (the channel is assumed to have unitary power) and the one-sided noise power spectral density. We also define Γ_u as the corresponding average SNR at the user location on the cooperative channel, and refer to the following correspondences: $\mathrm{SNR} = 10\log_{10}(\Gamma)$, $\mathrm{SNR_u} = 10\log_{10}(\Gamma_\mathrm{u})$, and $\Delta_\mathrm{SNR} = 10\log_{10}(\Gamma_\mathrm{u}/\Gamma)$.

The derivation of the BER has been shown in [16], while the derivation of the OP is shown in the Appendix.

3. The cooperative scheme

Consider a CG composed of J cooperative users, each transmitting on a different orthogonal channel. The system is synchronous and sends packets of $L_f = L_p + L_d$ symbols. Each packet is made of a "pilot frame" with L_p pilot symbols, and a "data frame" with $L_d = NL$ data symbols; the data frame is made of N subframes, each with L symbols. L also represents the number of times per frame that DBLC is applied. We use the following notation:

$$\boldsymbol{s}^{(j)} = \left(\boldsymbol{p}^{(j)\mathrm{T}}, \boldsymbol{d}^{(j)\mathrm{T}}\right)^\mathrm{T}, \qquad \boldsymbol{d}^{(j)} = \left(\boldsymbol{x}^{(j)}(1)^\mathrm{T}, \ldots, \boldsymbol{x}^{(j)}(N)^\mathrm{T}\right)^\mathrm{T},$$

$$\boldsymbol{p}^{(j)} = \left(p_1^{(j)}, \ldots, p_{L_p}^{(j)}\right)^\mathrm{T},$$

$$\boldsymbol{x}^{(j)}(n) = \left(x_1^{(j)}(n), \ldots, x_L^{(j)}(n)\right)^\mathrm{T},$$

where, referring to the jth user, $\boldsymbol{s}^{(j)}$, $\boldsymbol{p}^{(j)}$, and $\boldsymbol{d}^{(j)}$ denote a packet, a pilot frame, and a data frame, respectively, $\boldsymbol{x}^{(j)}(n)$ is the nth subframe, $p_\ell^{(j)}$ is the ℓth pilot symbol of the pilot frame, and $x_\ell^{(j)}(n)$ is the ℓth data symbol of the nth subframe. Symbols of the data frame convey information from KL source symbols via a DLBC of rate $R = K/N$; thus the overall rate, including both coding and pilot overhead, is $\rho = \frac{KL}{L_p+NL}$.

We consider the following model for quasi-static scenarios:

$$r_\ell^{(i,j)} = \sqrt{\rho\mathcal{E}_b}\,h^{(i,j)}s_\ell^{(j)} + w_\ell^{(i,j)}, \tag{6}$$

where, referring to the link from jth user to ith user (with $i = 0$ denoting the BS), $h^{(i,j)}$ represents the channel gain (constant for the transmission of the whole packet), $s_\ell^{(j)}$ denotes the ℓ symbol transmitted by the jth user, $r_\ell^{(i,j)}$ denotes the corresponding signal received by the jth user, and $w_\ell^{(i,j)}$ is the additive noise corrupting the reception. We use an analogous notation (to those used for the transmitted signals) for the received signals obtained via Eq. (6):

$$\boldsymbol{r}^{(i,j)} = \left(\boldsymbol{q}^{(i,j)\mathrm{T}}, \boldsymbol{g}^{(i,j)\mathrm{T}}\right)^\mathrm{T},$$

$$\boldsymbol{g}^{(i,j)} = \left(\boldsymbol{y}^{(i,j)}(1)^\mathrm{T}, \ldots, \boldsymbol{y}^{(i,j)}(N)^\mathrm{T}\right)^\mathrm{T},$$

$$\boldsymbol{q}^{(i,j)} = \left(q_1^{(i,j)}, \ldots, q_{L_p}^{(i,j)}\right)^\mathrm{T},$$

$$\boldsymbol{y}^{(i,j)}(n) = \left(y_1^{(i,j)}(n), \ldots, y_L^{(i,j)}(n)\right)^\mathrm{T}.$$

More specifically, communications to the BS happen through a two-step process: (i) pilot frames are sent by each user and received by the BS as well as other users exploiting the orthogonality of the channels; thus the BS and the users perform channel estimation for the various links in the CG; (ii) data frames are obtained iteratively applying L times a DLBC of rate R among the users of the CG. For channel estimation, from Eq. (6), applying linear MMSE estimation and the matrix inversion lemma, we get

$$\hat{h}^{(i,j)} = \frac{1}{\boldsymbol{p}^{(j)\mathrm{T}}\boldsymbol{p}^{(j)} + \eta_o/2}\boldsymbol{p}^{(j)\mathrm{T}}\boldsymbol{q}^{(i,j)}.$$

For cooperative transmission, for each $\ell = 1, \ldots, L$, the corresponding symbols from the N subframes ($x_\ell^{(1)}(1), \ldots, x_\ell^{(J)}(1); \ldots; x_\ell^{(1)}(N), \ldots, x_\ell^{(J)}(N)$) implement a DLBC, as explained in [16], with the BS recovering information from each group: ($y_\ell^{(0,1)}(1), \ldots, y_\ell^{(0,J)}(1); \ldots; y_\ell^{(0,1)}(N), \ldots, y_\ell^{(0,J)}(N)$), exploiting the decomposition of the matrix $\boldsymbol{B}$ into matrices $\boldsymbol{A}$ (in Section 2.1) for computational savings. For instance, referring to the basic example ($J = 3$ and $R = 2/3$), we have the algorithm

$$\begin{cases} x_\ell^{(j)}(1) = \mathrm{BPSK}(b_\ell^{(j)}(1)) \\ x_\ell^{(j)}(2) = \mathrm{BPSK}(b_\ell^{(j)}(2) + \hat{c}_\ell^{(j,j-1)}(1)) \\ x_\ell^{(j)}(3) = \mathrm{BPSK}(\hat{c}_\ell^{(j,j-1)}(2) + \hat{c}_\ell^{(j,j-2)}(1)), \end{cases}$$

where $\hat{c}$ refers to the bit corresponding to the estimate of the symbol x, $b + c$ is a modulo-2 sum, $j-1$ and $j-2$ are circular subtractions. The BS then applies the decoding algorithm for DLBC to the symbols ($y_\ell^{(0,1)}(1), y_\ell^{(0,2)}(1), y_\ell^{(0,3)}(1)$; $y_\ell^{(0,1)}(2)$, $y_\ell^{(0,2)}(2)$, $y_\ell^{(0,3)}(2)$; $y_\ell^{(0,1)}(3)$, $y_\ell^{(0,2)}(3)$, $y_\ell^{(0,3)}(3)$), building three groups: $\{y_\ell^{(0,1)}(1), y_\ell^{(0,2)}(2), y_\ell^{(0,3)}(3)\}$, $\{y_\ell^{(0,2)}(1), y_\ell^{(0,3)}(2), y_\ell^{(0,1)}(3)\}$, and $\{y_\ell^{(0,3)}(1), y_\ell^{(0,1)}(2), y_\ell^{(0,2)}(3)\}$.

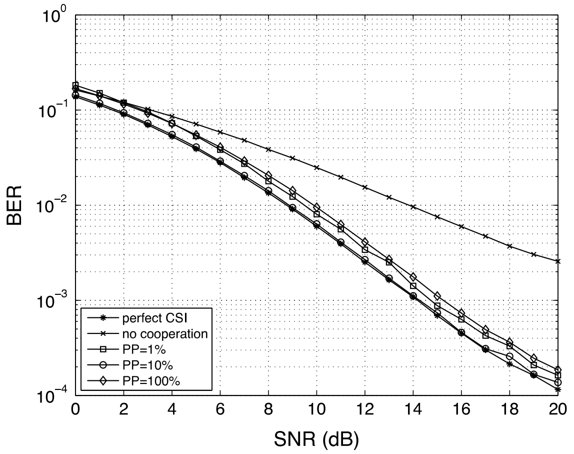

Fig. 1. Effect of PP on BER performance in the presence of cooperation, when perfect decoding is assumed at the user location. The reference curves represent perfect CSI at the BS in the absence and presence of cooperation.

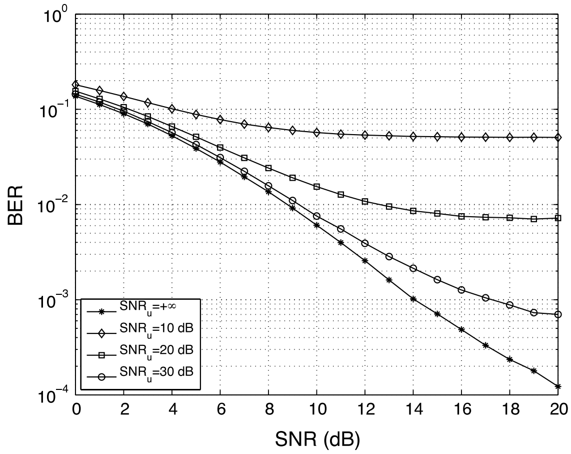

Fig. 2. Effect of channel estimation and decoding errors at the user location. First scenario: SNR_u is constant w.r.t. the SNR. The reference curve represents perfect decoding at the user location ($\text{SNR}_\text{u} = +\infty$).

4. Simulations

Computer simulations have been performed with Matlab. BPSK modulation with uniform *a priori* symbol distribution is assumed at the user location; linear MMSE channel estimation is performed both at the user location and the BS; ML decoding at the BS is performed by selection of the minimum distance in the signal-space constellation. Channels are statistically i.i.d. with constant coefficients within the frame, according to Rayleigh fading statistics [19]. It is worth noticing that in the following the SNR takes into account the power spent on pilot transmission; thus R is to be replaced with ρ in the expression of Γ.

First, the effect on the performance of PP, defined as the ratio L_p/L, is considered. Increasing the number of pilots has the twofold (positive–negative) effect of making the channels estimates more reliable and introducing a rate loss in the system. Fig. 1 shows the performance for $L = 500$ with $L_p = 5,\ 50,\ 500$, i.e. PP $= 1\%,\ 10\%, 100\%$, when perfect decoding is assumed at the user location. PP $=$ 10% represents the best choice, with 1 dB gain w.r.t. PP $= 1\%,\ 100\%$, and it has been confirmed for various choices of L (not shown here for brevity). Fig. 1 also shows, as reference terms, the performance of the system in the case of perfect decoding at the user location and perfect CSI at the BS, and the performance of an analogous system without cooperation among users. It is apparent how linear MMSE channel estimation allows one to achieve perfect CSI performance, and how the diversity gain due to cooperation is confirmed by the different slopes.

Second, the effects of decoding errors and channel estimation at both the user location and the BS are considered. Two scenarios are analyzed via simulations: (a) the quality on cooperative channels is fixed independently of the quality of the channels to the BS; (b) the quality on cooperative channels scales proportionally with the quality of the channels to the BS. The two cases assume constant SNR_u and Δ_{SNR}, respectively, w.r.t. the SNR. The former may be considered as a representation of a set of static users with fixed transmission power and with the BS moving towards them; the latter may be considered as a representation of a set of static users with increasing transmission power and static BS.

Fig. 2 shows the performance of the first scenario. It is apparent the presence of an error floor depending on the quality of the channels among users: if the quality of the channels to the BS improves without corresponding improvement of the quality of the channels among users, the system does not follow the expected performance. The reason behind the error floor is found in the suboptimal receiver; optimum receivers for relay channels are presented in [20,21]. As shown in Section 2, the proposed receiver assumes a decomposition of the signal space that provides a significant reduction of complexity. However, such a decomposition refers to the case of error-free decoding from cooperative users; thus the receiver cannot take into account errors at the user location. If the quality of the channels among users is poor, the error-free assumption at the receiver is unrealistic and the result is the error floor. Channels among cooperative users are required to have $\text{SNR}_\text{u} = 20 - 30$ dB, otherwise errors among cooperative users would overcome the benefits from diversity gain. Alternatively, cooperation may be used only when error-free decoding happens at the cooperative user, with users switching to non-cooperative transmissions in the opposite case. Receivers taking into account such a hybrid transmission mode have been designed in [13,15]; however, they rely heavily on a flagbit to make the BS aware of the current transmission mode.

In contrast, Fig. 3 shows the performance of the second scenario, in which the diversity gain is kept, and perfect CSI performance is practically achieved when $\Delta_{\text{SNR}} = 20$ dB.

5. Conclusion

The performance of DLBC has been presented in terms of BER and OP. A packet-based communication scheme

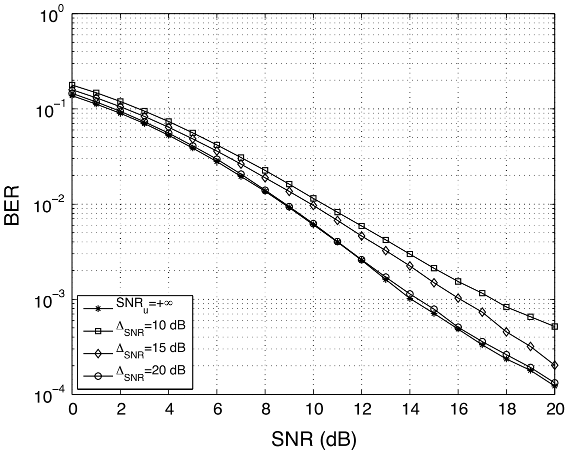

Fig. 3. Effect of channel estimation and decoding errors at the user location. Second scenario: Δ_{SNR} constant w.r.t. the SNR. The reference curve represents perfect decoding at the user location ($SNR_u = +\infty$).

using DLBC has been described, and its performance shown via numerical simulations. The effects of the number of pilots, decoding errors, and channel estimation errors have been studied, with a focus on a static set of users transmitting to a mobile or static BS. The two scenarios emphasize the cases in which the cooperative channels improve or not with the channel to the BS: in the first case the advantage of the cooperative system saturates, as shown by the presence of an error floor in the performance, while in the second case the cooperative system is always effective.

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Appendix. Derivation of the outage probability

Defining an outage event as $\left\{\log\left(1+\Gamma|h|^2\right)<R\right\}$, it is straightforward to obtain Eq. (3). The OP for DLBC in Rayleigh i.i.d. channels is

$$P_o = \Pr\left(\sum_{n=1}^{N}\log\left(1+\Gamma|h_n|^2\right)<R\right)=\varphi(N;\Gamma,2^R),$$

where

$$\varphi(N;\gamma,r)=\int_{\mathcal{D}(\gamma,r)}\exp\left(-\sum_{n=1}^{N}\eta_n\right)\mathrm{d}\eta_1\ldots\mathrm{d}\eta_N,$$

$$\mathcal{D}(\gamma,r)=\left\{(\eta_1,\ldots,\eta_N)\in\mathcal{R}^N:\prod_{n=1}^{N}(1+\gamma\eta_n)<r\right\}.$$

Considering, for the sake of simplicity, the case with $K=2$ and $N=3$ (the basic example with $R=2/3$), and using the series expansion $\exp(x)=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2}+\mathcal{O}(x^3)$ along the same lines as [11], we have

$$\varphi(3;\gamma,r) = \int_0^{a_1}\mathrm{d}\eta_1\int_0^{a_2}\mathrm{d}\eta_2\int_0^{a_3}\mathrm{d}\eta_3\ \exp(-\eta_1-\eta_2-\eta_3),$$

where we have defined

$$a_1=\frac{r-1}{\gamma},\qquad a_2=\frac{1}{\gamma}\left(\frac{r}{1+\gamma\eta_1}-1\right),$$

$$a_3=\frac{1}{\gamma}\left(\frac{r}{(1+\gamma\eta_1)(1+\gamma\eta_2)}-1\right).$$

Simple mathematics then gives $\varphi(3;\gamma,r)=I_1-I_2-I_3$, with

$$I_1=1-\exp(-a_1)=a_1+\frac{a_1^2}{2}+\mathcal{O}\left(\frac{1}{\gamma^3}\right),$$

$$\begin{aligned}I_2 &= \int_0^{a_1}\mathrm{d}\eta_1\exp(-\eta_1-a_2)\\ &= a_1-\frac{a_1^2}{2}-r\log(r)\frac{1}{\gamma^2}+\frac{a_1}{\gamma}+\mathcal{O}\left(\frac{1}{\gamma^3}\right),\end{aligned}$$

$$\begin{aligned}I_3 &= \int_0^{a_1}\mathrm{d}\eta_1\int_0^{a_2}\mathrm{d}\eta_2\exp(-\eta_1-\eta_2-a_3)\\ &= r\log(r)\frac{1}{\gamma^2}-\frac{a_1}{\gamma}+\mathcal{O}\left(\frac{1}{\gamma^3}\right),\end{aligned}$$

and finally $\varphi(3;\gamma,r)\approx\frac{(r-1)^2}{\gamma^2}$, or analogously, in the general case, Eq. (5).

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